PUZZLES VERSUS MATHS QUESTIONS

by **Rob Eastaway**

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In the last few years, puzzles have come firmly back into fashion. It all started with a book of teasers set by the secretive GCHQ, which turned out to be a runaway Christmas bestseller. Numerous successful puzzle books have followed, written by Alex Bellos, the UKMT and many others, demonstrating that the public has an almost insatiable appetite for mental stimulation.

Over the years, one of the highest profile forums for puzzles was BBC Radio 4's *Today* programme. At 6:50 am every weekday morning, over a million listeners heard a daily puzzle read out (sometimes rather sneeringly, it must be said) by one of the presenters. It was the *Today* programme puzzle, in particular, that re-engaged me with a debate that I first raised at a teacher conference over twenty years ago.

What *is* a puzzle?
How does it differ from a mathematics question?
And does it matter?

For most people, there is a clear distinction between puzzles and mathematics questions. For example, is this a puzzle or a mathematics question?

Some explorers walk a mile south, a mile east and a mile north and discover they are back where they started. Then they shoot a bear. What colour is the bear?

I expect you think that was a puzzle, and if so, you are in the majority. (It's an old chestnut, but if you happen not to have heard it before, the answer is at the end of the article.)

At the other extreme, what about this?

Solve the equation:

$$\frac{x}{6} = 1 - \frac{(1-x)}{8}.$$

Almost everyone (from general public to mathematics teachers) regards this as a mathematics question, not a puzzle. So what's the difference? I like to ask mathematics teachers to reflect on why they think of some problems as puzzles and others as mathematics questions. Here are some of their descriptions of mathematics questions:

'They involve applying a known approach'
'Maths questions are dry and abstract'
'They are used to practise maths skills'
'The sort of thing you only do in maths lessons'

And here's another that came up in one discussion:

'A maths question is something that's only enjoyed by "maths people" '.

I'm not going to attempt to define who or what a "maths person" is, but I have no doubt that there is some line that separates those who enjoy tackling mathematical challenges *because they are mathematics*, from the vast majority who are not enticed by such problems unless they have some other redeeming feature.

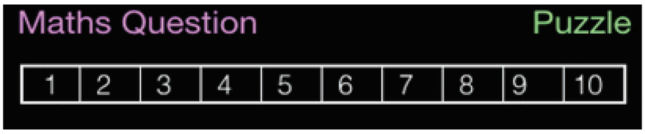
Contrast the above comments on mathematics questions with the descriptions that come back for puzzles:

'Puzzles involve tricks and surprises'
'Seem impossible until you have an aha moment'
'Require a lateral approach to solving it'
'They connect with the real world'
'Puzzles have an amusing story'

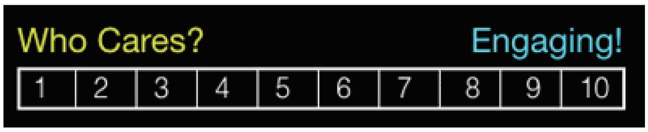
But the most common description that I hear about puzzles is that:

'They are fun'

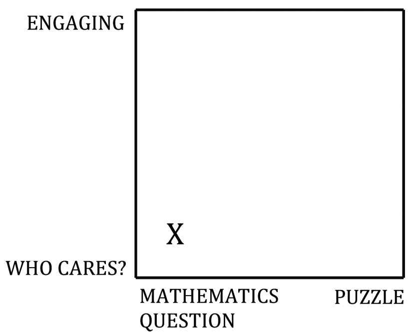
These are generalisations, of course. Not every mathematics question or puzzle has all of the features above, and nor does everyone agree on whether a particular example is a puzzle or a mathematics question. In fact, rather than a binary choice between 'mathematics question' and 'puzzle', there is really a spectrum. On a scale of 1 to 10, where 1 is definitively a mathematics question and 10 is a puzzle, a particular problem might rate as, say, a '3'.



There's another scale that interests me more. Some problems seem to capture the imagination of a wide audience, while others engage very few. I call this the *Who Cares?* scale. Everyone wants to tackle a 10/10 engaging problem. Only masochists or obsessives are bothered with tackling ones that rate as 1/10.



Using these two scales, we can now plot any problem on a 2-dimensional chart:



Where on the chart would you rate the following problem?

Amy boards a plane at Gatwick and flies 1860 miles in 150 minutes for a holiday. Assuming the plane flies at constant speed, how many miles has Amy flown in 3¾ hours?

This was posed as a *Today* programme 'puzzle'. Do my mathematics teacher audiences regard it as a puzzle? On balance, no. They put it at the Mathematics Question end of the spectrum. Many of them also regard it as a 'Who Cares?' problem, the average score being roughly where the X is on the diagram. (Outside the mathematics teacher bubble, this sort of problem generally rates as 100% 'Who cares?').

Why does the plane problem score so badly on engagement? There are several reasons: the 'story' behind the problem is dull; the mathematics involved is routine and rather uninspiring; and there's no satisfying revelation in the answer. About the only satisfaction to be had in a problem like this is the experience of getting it right.

Whether a problem rates as a 'mathematics question' or a 'puzzle' does not in itself matter, it's the engagement that is important. However, I believe the two are related. The more 'puzzly' a problem is, the more 'engaging' people tend to find it. (Especially non-mathematics people – see my definition above.) So if you want to engage more of your students, I would argue that the problems you pose to them need to be more puzzly.

How to make problems more engaging

To learn more about the link between puzzliness and engagement, I conducted an experiment with a group of students in the age range 13–17 at a school in Dorset. It wasn't the most scientific of experiments. These were relatively keen mathematicians – they had volunteered to spend some of their lunch break with me! Nonetheless, their responses still gave some insights into how students react to different problems.

I presented the group with several problems, and asked them to rate each problem on my puzzle and engagement scales. One problem that I gave them was a classic that I have been sharing with students and teachers for many years:

My dictionary has two volumes, A-M and N-Z which sit by each other on the shelf. (A-M on the left, N-Z on the right, as you would expect). Each volume is 5 cm thick, and the covers are 2 mm thick. I bookmark the words Aardvark and Zebra. How far apart are the two bookmarks?

The group rated this 4 as a puzzle (i.e. they saw it more as a mathematics question), and they gave it a low-ish 3.5 for engagement. That is, until they heard the solution.

Most of the students expected the answer to be 9.6 cm or thereabouts, and were astonished to discover that the correct answer is 4 mm. (Why? Because when you look at the two volumes on the shelf, the first page of the 'A-M' volume is on the right and the last page of 'N-Z' on the left, so aardvark and zebra are next to each other.)

Discovering the surprise solution changed the students' view of the problem – they now scored it 6 as a puzzle, and 6.5 for engagement. Which goes to show that the 'enjoyment' of a puzzle sometimes comes in discovering an unexpected solution, rather than in the problem or the working out. There's no fun to be had until you discover you were wrong.

But if we want to appeal to a wider audience, we can't rely on a puzzle's 'wow' factor coming at the end. The most engaging problems are ones that draw you in from the start. Take this one for example (which I first heard from a friend, Angus Walker):

How can Caesar take you from 3 to 47?

My student group scored this one as a 7 on the puzzle scale, and 6.5 for engagement. The problem is intriguing, a mystery, with no clear indication of what approach to take. If the initial intrigue wears off, you can boost the level of engagement by offering hints. (Here's one hint: write the number 3 as a word, in capitals.) The Caesar puzzle has two nice solutions to it, by the way, which you'll find at the end of the article.

It's important to have engagement at the start of the puzzle-solving process, so is it possible to take a dry 'mathematics question' and give it more appeal by making it more puzzly? Yes, though it takes skill to do this well. Here's an example:

$A + B = 78$,
$B + C = 69$,
$A + C = 137$.
Work out $A + B + C$.

My students rated this 1.5 for puzzliness and 3 for engagement. That's pretty low, but my guess is that a less-mathematical group would have rated it even lower).

One way in which mathematics problem setters and examiners try to make problems like this more appealing is by making A, B and C represent something physical in the real world. For example, this:

Alf and Bert together weigh 78kg. Bert and Charlie weigh 69kg. And Alf and Charlie weigh 137kg. What is the combined weight of Alf, Bert and Charlie?

Hmm. Even though A, B and C now represent people, the problem isn't much more engaging in this form. Who are Alf, Bert and Charlie, and why should we be interested in their combined weights? And how is it that we know the weights of them in pairs yet we don't already know their individual weights? My student group saw through this wheeze immediately. Their rating of it for puzzliness and engagement did move up the scale, but only by a tiny amount.

I now showed them the same problem in a different context – the one used by UKMT when the problem was first posed as a challenge question about 30 years ago.

Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off 78 kg. Then the nurse held the baby while I read off 69 kg. Finally I held the nurse while the baby read off 137 kg. What is the combined weight of all three?

My student group laughed. This time there is a *reason* why we don't know the individual weights – it's because babies can't weigh themselves. And the laugh that comes from the absurdity of the final weighing made them much more motivated to tackle the problem. They now viewed the problem as 4.5 for puzzliness (an increase of 3 points) and 5.5 for engagement (2½ points higher). The best puzzles have an engaging and plausible (or comedic) storyline, and a good punchline.

Finally, do you remember the 'mathematics question' that featured at the start of this article?

Solve the equation: $\frac{x}{6}=1-\frac{(1-x)}{8}$.

Here it is again, this time presented as a puzzle.

Tim left this card on the table:

$$\frac{\mathrm{X}}{6}=\mathrm{I}-\frac{(\mathrm{I}-\mathrm{X})}{8}$$

"I've worked out what x is!" said Bob, a minute later. Amy walked over and picked up the card. Five minutes later she announced: "I've found TWO solutions". Amy's solutions were different from Bob's. What were they?

What started as a routine algebra problem has turned into something much more intriguing. Can you solve it?

ANSWERS

1. The bear is white. The only place where you can go south, east and north in this way and see a bear is near to the North Pole.
2. If you (Caesar) shift the word THREE by four places, you get XLVII, which is 47. Alternatively, the letter C is number 3 in the alphabet, and if you add all the letters of CAESAR you get:

   3 + 1 + 5 + 19 + 1 + 18 = 47.
3. Bob's solution was x = 21. The formula was written on a card and when Amy picked up the card she must have looked at it upside down. The upside-down equation is

   $$\frac{8}{(x-1)}-1=\frac{9}{x}$$

   and has two solutions: x = 3 and x = –3.

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Author Rob Eastaway is an author, a puzzle adviser to *New Scientist* magazine, Director of *Maths Inspiration* and a former President of the Mathematical Association.
e-mail: **email@robeastaway.com**